Physics 323 Lecture Notes
Part I: Optics

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Contents

Chapter 1

Nature of light

1.1 Light – Wave or stream of particles?

Answer: Yes! As we'll see below, there is experimental evidence for both interpretations, although they seem contradictory.

1.1.1 What is a wave?

More familiar types of waves are sound, or waves on a surface of water. In both cases, there is a **perturbation** with a periodic spatial pattern which **propagates**, or travels in space. In the case of sound waves in air for example, the perturbed quantity is the pressure, which oscillates about the mean atmospheric pressure. In the case of waves on a water surface, the perturbed quantity is simply the height of the surface, which oscillates about its stationary level. Figure 1.1 shows an example of a wave, captured at a certain instant in time. It is simpler to visualize a wave by drawing the "wave fronts", which are usually taken to be the crests of the wave. In the case of Figure 1.1 the wave fronts are circular, as shown below the wave plot.

1.1.2 Evidence for wave properties of light

There are certain things that only waves can do, for example **interfere**. Ripples in a pond caused by two pebbles dropped at the same time exhibit this nicely: Where two crests overlap, the waves reinforce each other, but where a crest and a trough coincide, the two waves actually cancel. This is illustrated in Figure 1.2. If light is a wave, two sources emitting waves in a synchronized fashion[1] should produce a pattern of alternating bright and dark bands on a screen. Thomas Young tried the experiment in the early 1800's, and found the expected pattern.

The wave model of light has one serious drawback, though: Unlike other wave phenomena such as sound, or surface waves, it wasn't clear what the medium was that supported light waves. Giving it a name – the "luminiferous aether" – didn't help. James Clerk Maxwell's (1831 - 1879) theory of electromagnetism, however, showed that light was a wave in combined electric and magnetic fields, which, being force fields, didn't need a material medium.

[1] When two sources of waves oscillate in step with each other, they are said to be **coherent**. We will return to this when we study interference phenomena in greater detail.

1.1.3 Evidence for light as a stream of particles

One of the earliest proponents of the idea that light was a stream of particles was Isaac Newton himself. Although Young's findings and others seemed to disprove that theory entirely, surprisingly other experimental evidence appeared at the turn of the 20th. century which could only be explained by the particle model of light! The **photoelectric effect**, where light striking a metal dislodges electrons from the metal atoms which can then flow as a current earned Einstein the Nobel prize for his explanation in terms of **photons**.

We are forced to accept that both interpretations of the phenomenon of light are true, although they appear to be contradictory. One interpretation or the other will serve better in a particular context. For our purposes, in understanding how optical instruments work, the wave theory of light is entirely adequate.

1.2 Features of a wave

We'll consider the simple case of a **sine wave** in 1 dimension, as shown in Figure 1.3. The distance between successive wave fronts is the **wavelength**.

As the wave propagates, let us assume in the positive x direction, any point on the wave pattern is displaced by dx in a time dt (see Figure 1.4). We can speak of the **propagation speed** of the wave

$$v = \frac{dx}{dt} \tag{1.1}$$

As the wave propagates, so do the wavefronts. A stationary observer in the path of the wave would see the perturbation oscillate in time, periodically in "cycles". The duration of each cycle is the **period** of the wave, and the number of cycles measured by the observer each second is the **frequency**[2]. There is a simple relation between the wavelength λ, frequency f, and propagation speed v of a wave:

$$v = f\lambda \tag{1.2}$$

Electromagnetic waves in vacuum always propagate with speed $c = 3.0 \times 10^8$ m/s. In principle, electromagnetic waves may have any wavelength, from zero to arbitrarily long. Only a very narrow range of wavelengths, approximately 400 - 700 nm, are visible to the human eye. We perceive wavelength as colour; the longest visible wavelengths are red, and the shortest are violet. Longer

[2] The SI unit of frequency is the Hertz (Hz), equivalent to s^{-1}.

than visible wavelengths are infrared, microwave, and radio. Shorter than visible wavelengths are ultraviolet, X rays, and gamma rays.

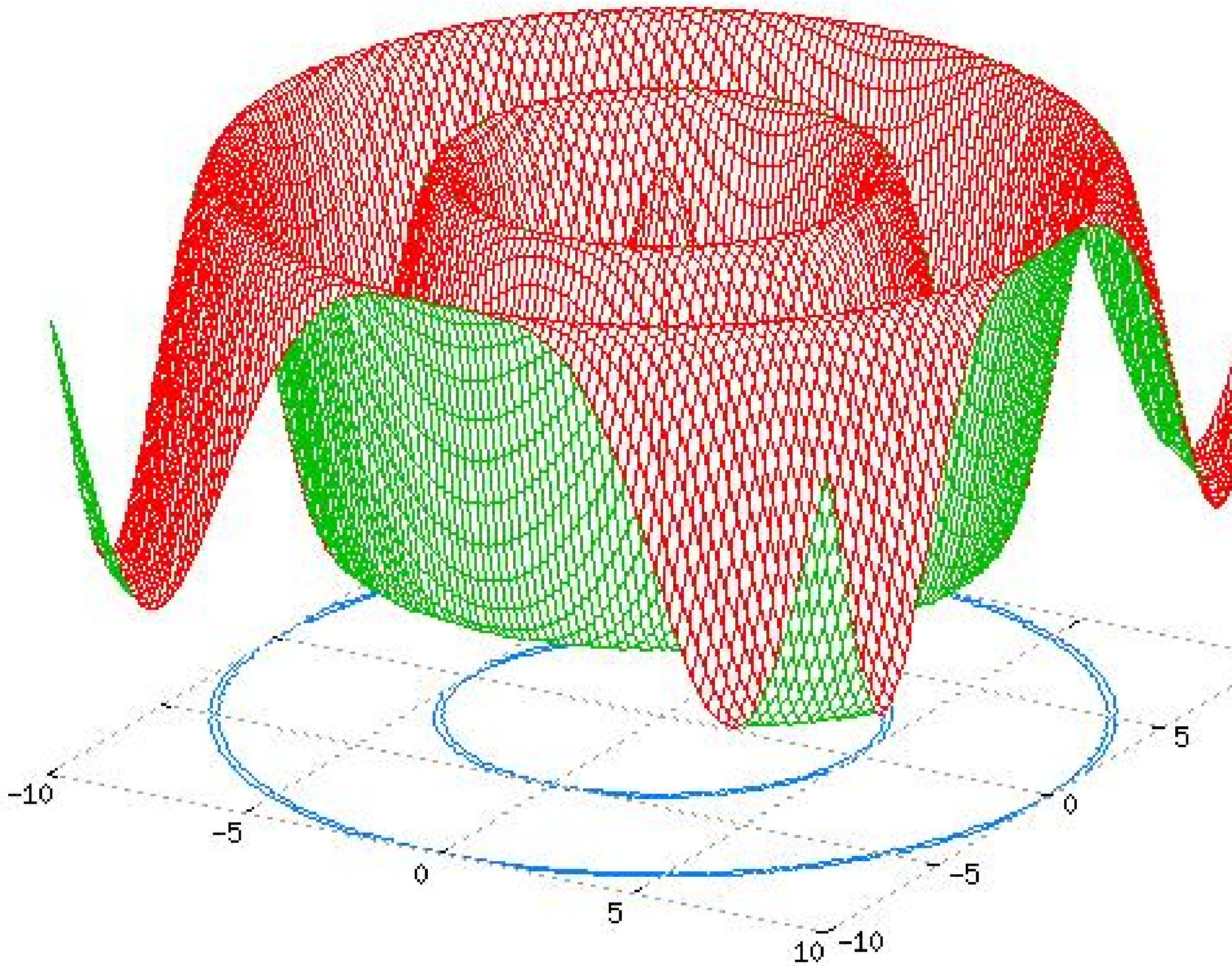

Figure 1.1: A wave

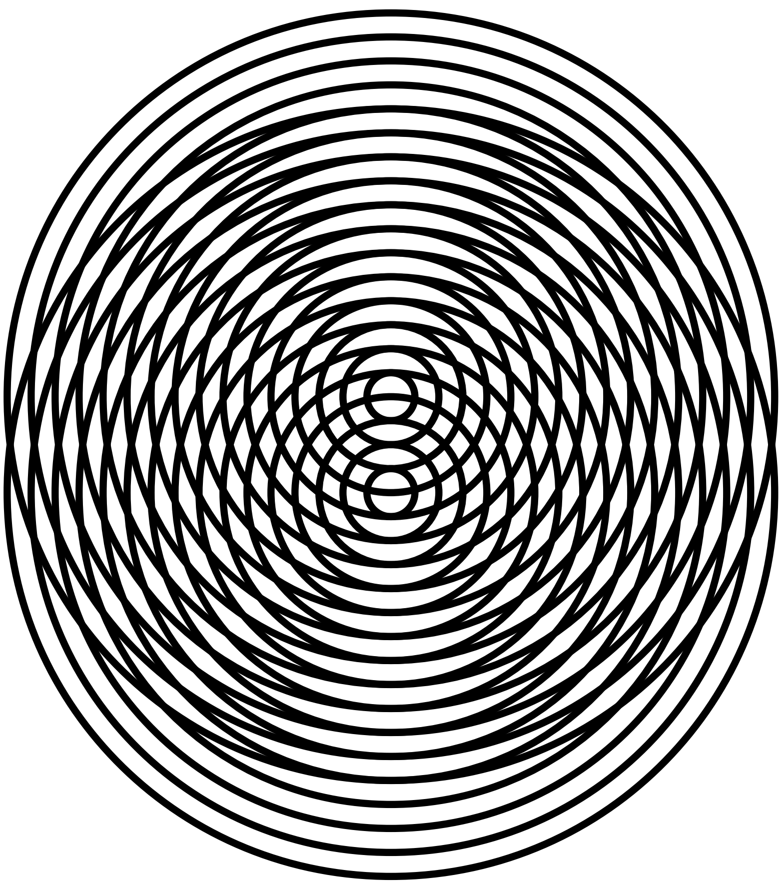

Figure 1.2: Interference

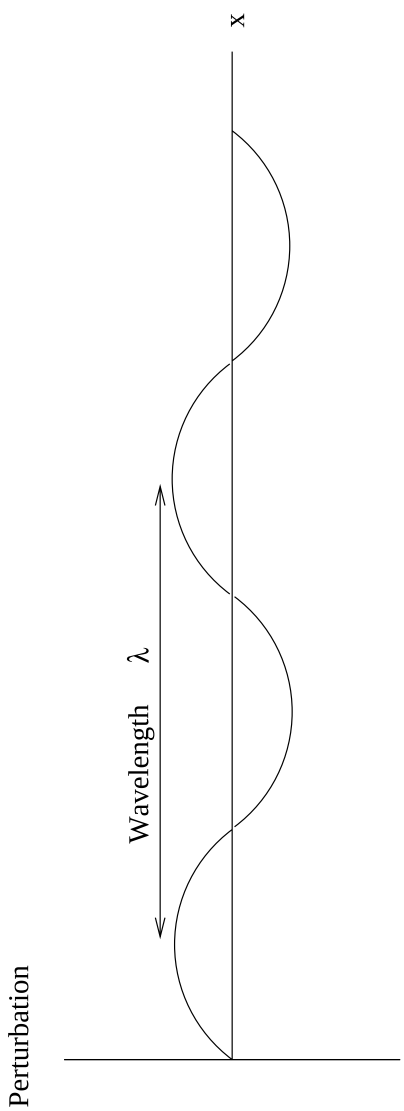

Figure 1.3: A sine wave

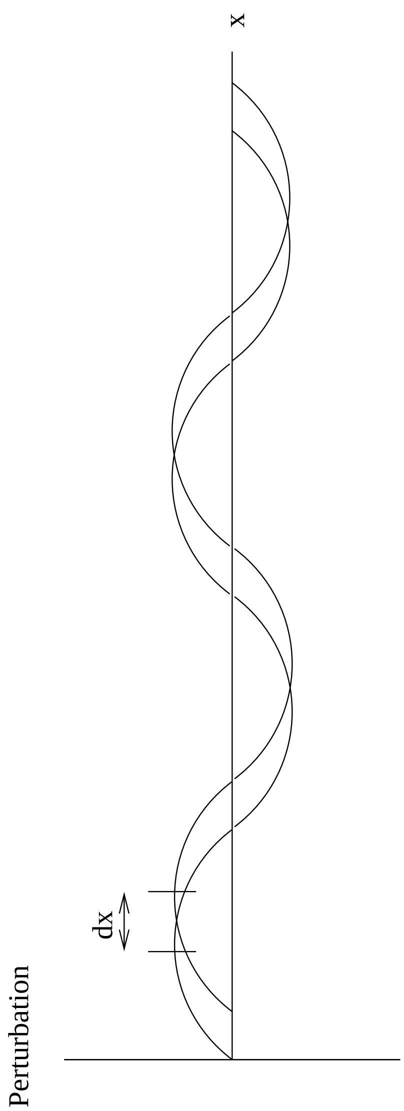

Figure 1.4: Wave propagation

Chapter 2

Propagation of light

2.1 Huygens' Principle

In the 1670's **Christian Huygens** proposed a mechanism for the propagation of light, nowadays known as **Huygens' Principle**:

> All points on a wavefront act as sources of new waves, and the envelope of these secondary waves constitutes the new wavefront.

Huygens' Principle states a very fundamental property of waves, which will be a useful tool to explain certain wave phenomena, like refraction below.

2.2 Refraction

When light propagates in a transparent material medium, its speed is in general less than the speed in vacuum c. An interesting consequence of this is that a light ray will change direction when passing from one medium to another. Since the light ray appears to be "broken", the phenomenon is known as **refraction**.

Huygens' Principle explains this nicely. See Figure 2.1. A plane wavefront (dashed line) approaches the interface between two media. At one end, a new wavefront propagates outwards reaching the interface in a time t according to Huygens' principle, so its radius is $v_1 t$. At the other end a new wavefront is propagating into medium 2 more slowly, so that in the same time t it has reached a radius $v_2 t$. Now consider the **angle of incidence** θ_i and the **angle of refraction** θ_r between the incident wavefront and the interface, and between the refracted wavefront and the interface. From the figure we see that

$$\sin\theta_i = \frac{v_1 t}{x} \text{ and } \sin\theta_r = \frac{v_2 t}{x} \Rightarrow \frac{\sin\theta_i}{\sin\theta_r} = \frac{v_1}{v_2} \tag{2.1}$$

This result is usually written in terms of the **index of refraction** of each medium, which is defined as

$$n = \frac{c}{v} \tag{2.2}$$

so that

$$n_1 \sin\theta_i = n_2 \sin\theta_r \tag{2.3}$$

a result which is known as **Snell's law**.

Refractive indices are greater than 1 (only vacuum has an index of 1). Water has an index of refraction of 1.33; diamond's index of refraction is high, about 1.5. It is tempting to think that the

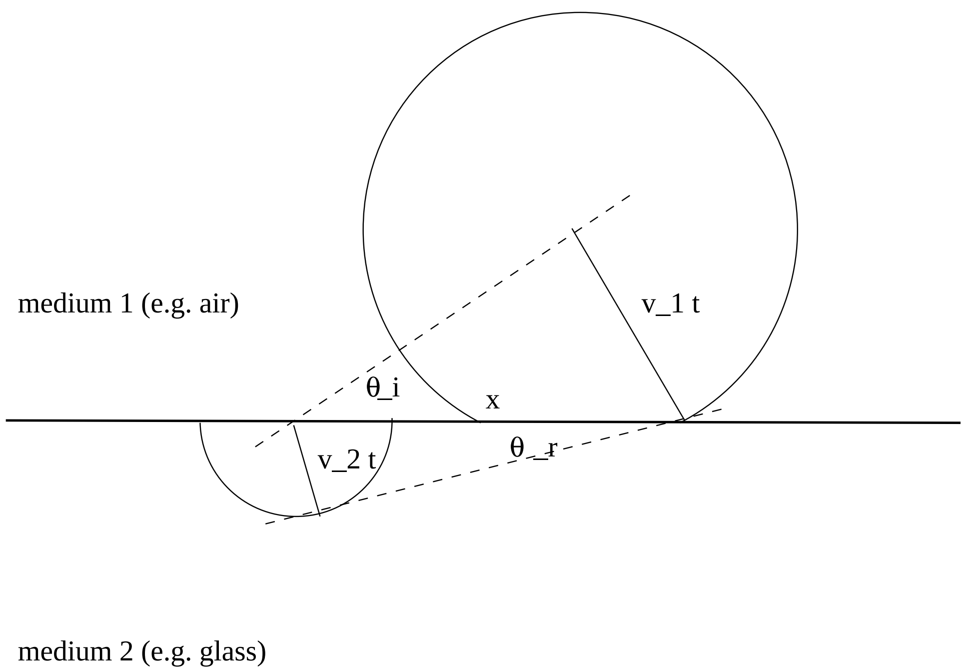

Figure 2.1: Refraction

index of refraction might be associated with the density of the material, but that is not the case. The idea lingers in the term **optical density**, a property of a material that the index of refraction measures.

2.3 Total internal reflection

One important consequence of Snell's law of refraction is the phenomenon of total internal reflection. If light is propagating from a more dense to a less dense medium (in the optical sense), i.e. $n_1 > n_2$, then $\sin\theta_r > \sin\theta_i$. Since $\sin\theta \leq 1$, the largest angle of incidence for which refraction is still possible is given by

$$\sin\theta_i \leq \frac{n_2}{n_1} \tag{2.4}$$

For larger angles of incidence, the incident ray does not cross the interface, but is reflected back instead. This is what makes optical fibres possible. Light propagates inside the fibre, which is made of glass which has a higher refractive index than the air outside. Since the fibre is very thin, the light beam inside strikes the interface at a large angle of incidence, large enough that it is reflected back into the glass and is not lost outside. Thus fibres can guide light beams in any desired direction with relatively low losses of radiant energy.

Chapter 3

Images

3.1 Images

An **optical system** creates an **image** from an **object**. For example, a slide projector shows an image of a slide on a screen. There are two types of images, **real** and **virtual**.

Since an extended object may be treated as a collection of point sources of light, we are specially interested in the images of **point objects**.

3.1.1 Real images

The formation of a real image is shown schematically in Figure 3.1. A point object emits light rays

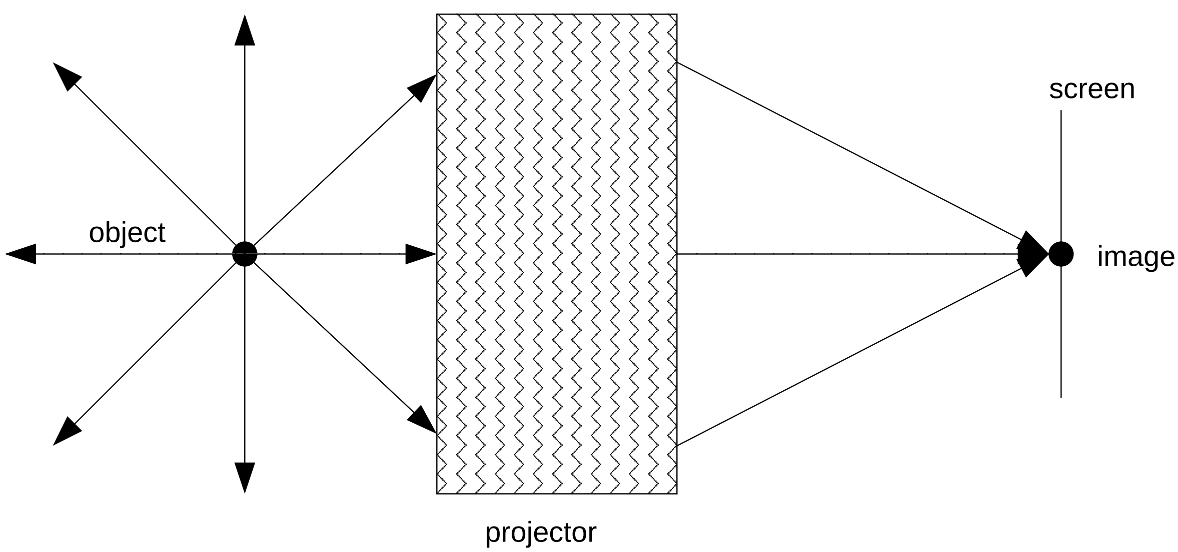

Figure 3.1: Formation of a real image

in all directions. Some are redirected by the optical elements in the projector *so that they converge to a point image.* If a screen is placed there, the image may be seen as the light concentrated there is scattered by the screen.

3.1.2 Virtual images

The reflection from a plane mirror is a good example of a virtual image. See Figure 3.2. The rays reflected by the mirror *seem to come from a point behind the mirror.* When those rays enter the

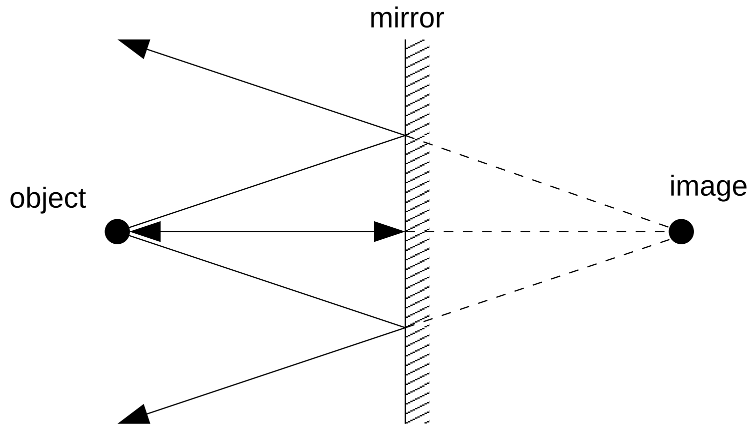

Figure 3.2: Virtual image formed by a plane mirror

eye of an observer or the objective of a camera, they will be seen as coming from a point. In that sense, we see the image of the object, but there is of course nothing actually there. If we placed a screen behind the mirror, nothing would be projected on it.

3.2 Curved mirrors

Curved mirrors are a key element of telescopes. They are usually **parabolic** in cross-section, for reasons to be discussed below. A **spherical** mirror is a good approximation if the curvature is low. A key property which is satisfied exactly by a parabolic mirror and approximately by a spherical one is the ability to focus a beam of light parallel to the **optical axis** – the axis of symmetry of the mirror – to a point, known as the mirror's **focal point** (see Figure 3.3).

3.3 Ray tracing with mirrors

To locate an image formed by a curved mirror, particular **auxiliary rays** from the object may be constructed. Consider the situation shown in Figure 3.4. Ray (1) from the object is parallel to the

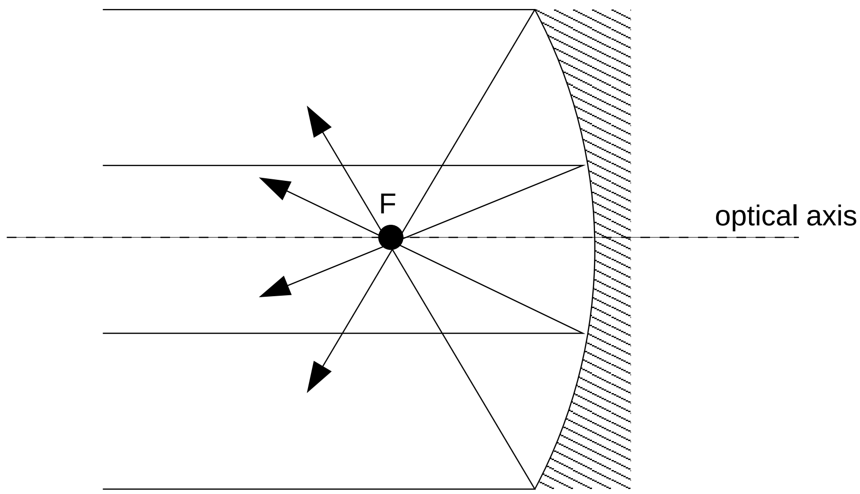

Figure 3.3: Focal point of a curved mirror

optical axis, and therefore passes through the focal point F after reflection. Ray (2) passes through F, and therefore is reflected parallel to the axis, according to the principle of reversibility of light. Ray (3) is reflected at the vertex of the mirror, so the reflected ray is symmetrical to the incoming ray with respect to the axis of the mirror. The image is formed at the intersection of the three rays. In fact, to locate the image we only need to construct two of the three possible auxiliary rays: Where they intersect is where the image is formed.

If we are dealing with an extended object, the whole image may be constructed this way. In the present example we can characterize the image as **real**, **inverted** (as opposed to **upright**), and **enlarged** (as opposed to **reduced**).

3.4 The mirror equation

The location of the image may be calculated from the position of the object and of the mirror's focal point by means of the **mirror equation**, which we shall derive shortly. These positions are measured by the following coordinates, illustrated in Figure 3.4: the **object distance** p measured along the axis from the **vertex** of the mirror, where the axis intersects the mirror; the **image distance** i, and the **focal length** f, measured in the same way. By convention, we draw the diagram so that the light is incident from the left, and all three lengths are counted as positive towards the left as indicated in the figure.

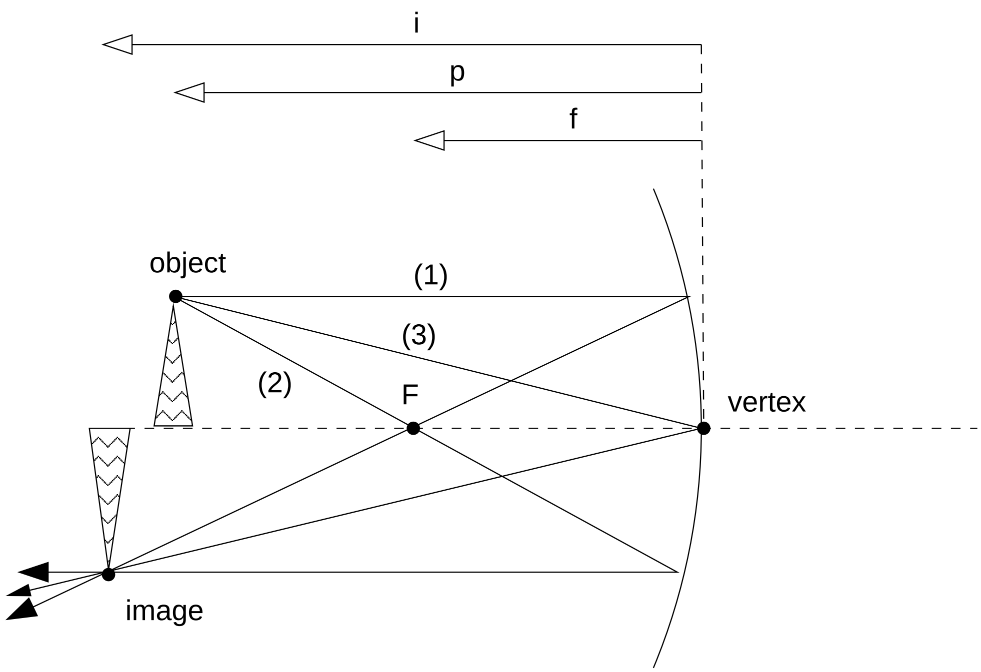

Figure 3.4: Image formation by a curved mirror

Our mirror equation presupposes that *the curvature of the mirror is very small*, which is true if the object is relatively small and close to the optical axis. In that case, we can draw the mirror as approximately flat. The situation is depicted in Figure 3.5. The triangles $\triangle OPF$ and $\triangle FQI$ are similar (check this). This means that the following ratios are equal:

$$\frac{p-f}{f} = \frac{f}{i-f} \tag{3.1}$$

After some manipulation, this expression reduces to

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \tag{3.2}$$

Exercise 3.4.1 *Derive equation (3.2) from equation (3.1).*

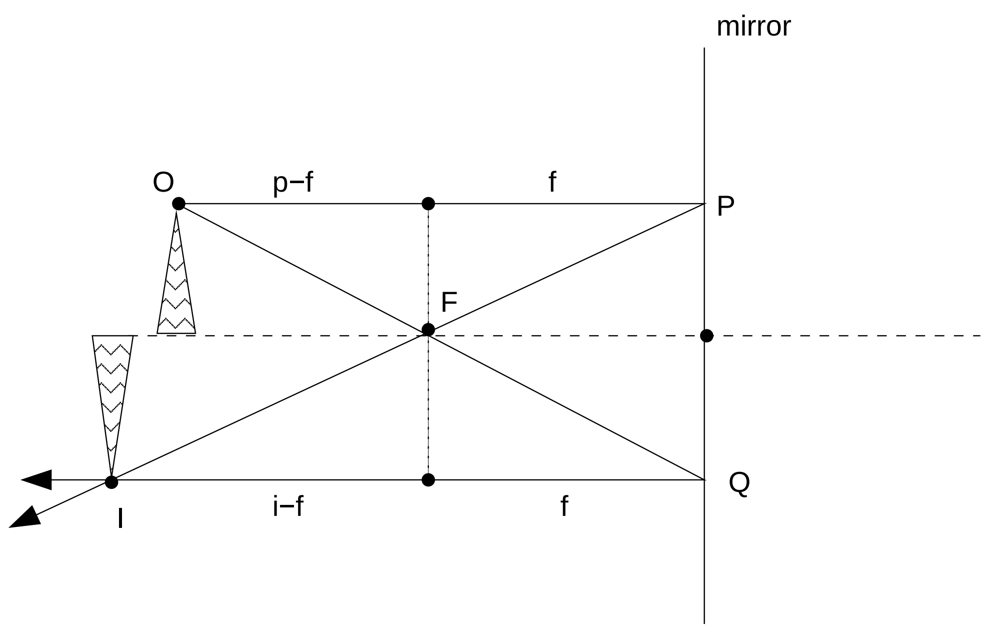

Figure 3.5: Derivation of the mirror equation

Chapter 4

Lenses

4.1 Introduction

Mirrors, which form images by reflection, are important components of telescopes; lenses, which form images by refraction, are also important components of many optical systems, including (refracting) telescopes. Next, we shall see how an image is formed as light rays from an object pass through two interfaces, generally air/glass and glass/air. Our first task is to locate the image of a point after passing through a single spherical interface.

4.2 Refraction at a spherical interface

We will restrict ourselves to those cases where the curvature of the interface is very small, so that we can represent it as a flat surface (albeit with a finite radius of curvature!) as shown in Figure 4.1. In the figure, a point object at O emits a ray of light along the optical axis, and another ray

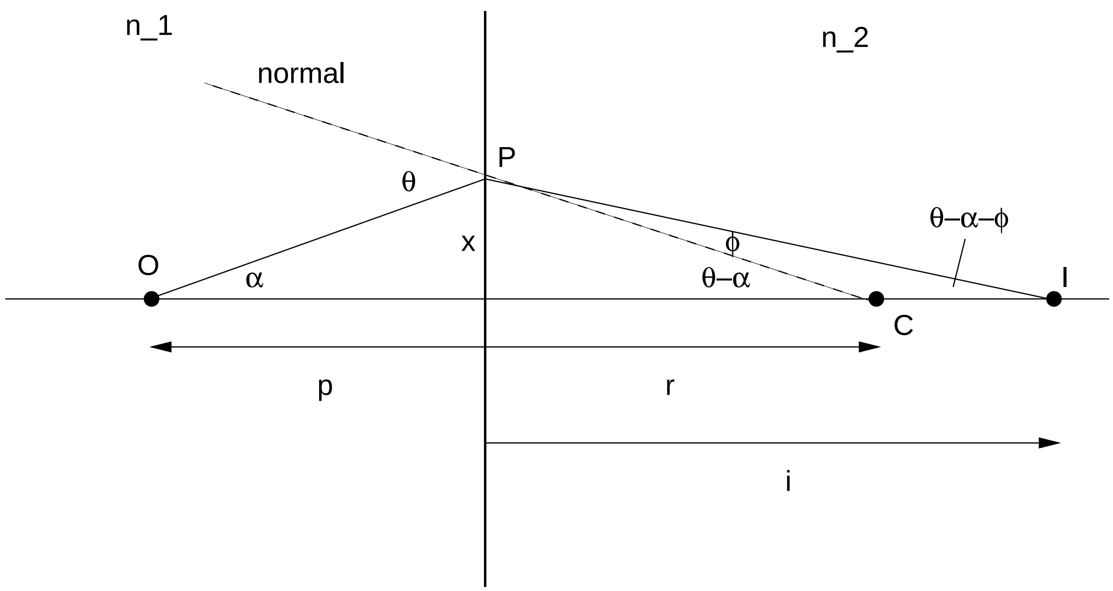

Figure 4.1: Refraction at a spherical surface

of light which is refracted at the interface and intersects the first one to form an image at I. The radius of curvature of the interface is r; as usual, the object distance to the interface is p and the image distance is i, butnote the following important caveat:

> *The sign convention for refraction is different from the one for mirrors: object distances are counted as positive when the object is in front of the interface, but image distances are positive when the image is formed behind the interface. The radius of curvature follows the same convention as the image distances.*

In the case of Fig. 4.1, the surface is convex, so the centre of curvature C lies to the right, and r is positive.

For the oblique ray, the incidence angle is θ and the refracted angle is ϕ. Then, by the exterior angle theorem, $\angle PCO = \theta - \alpha$ and $\angle PIC = \theta - \alpha - \phi$.

In the small angle approximation (see Appendix A), Snell's law becomes

$$n_1\theta = n_2\phi \tag{4.1}$$

and we can also approximate the angles as follows:

$$\alpha = \frac{x}{p} \tag{4.2}$$

$$\theta - \alpha = \frac{x}{r} \Rightarrow \theta = \frac{x}{p} + \frac{x}{r} \tag{4.3}$$

$$\theta - \alpha - \phi = \frac{x}{i} \Rightarrow \phi = \frac{x}{r} - \frac{x}{i} \tag{4.4}$$

and substituting θ and ϕ in Snell's law, we get after cancelling x

$$\frac{n_1}{p} + \frac{n_1}{r} = \frac{n_2}{r} - \frac{n_2}{i} \tag{4.5}$$

which can be rearranged more meaningfully to

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r} \tag{4.6}$$

If the light is passing from air of refractive index $n_1 = 1$ to glass of index $n_2 = n$, equation 4.6 becomes

$$\frac{1}{p} + \frac{n}{i} = \frac{n-1}{r} \tag{4.7}$$

4.3 A lens

4.3.1 Locating the image

In a lens, there are two consecutive refractions, one from air to glass, and then from the glass back into the air. Figure 4.2 shows the process. Applying eq. (4.6) to the first refraction, we get

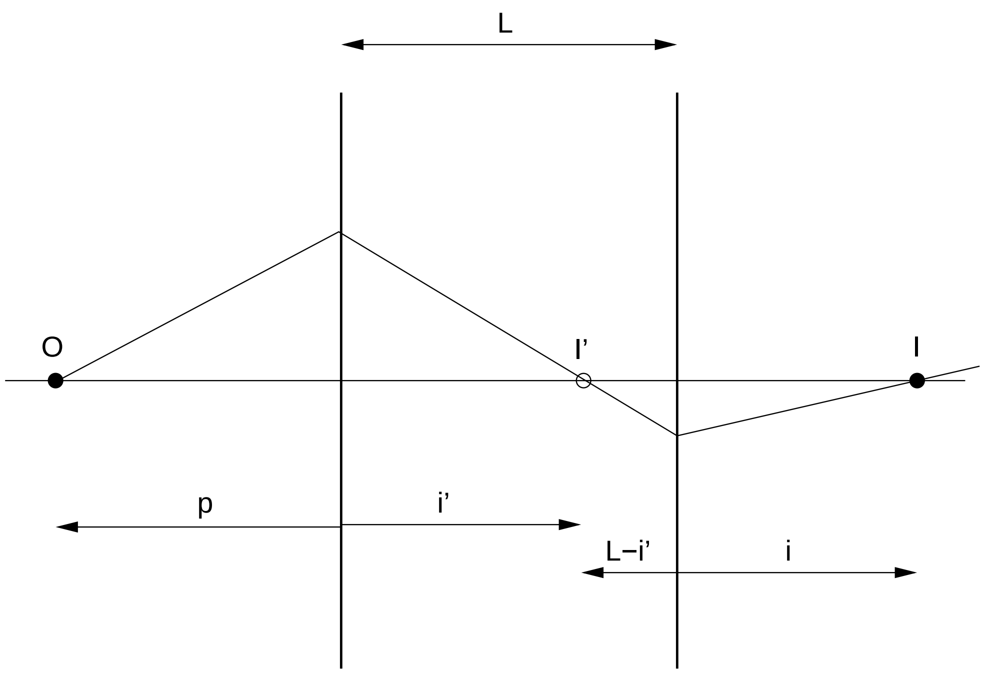

Figure 4.2: Two consecutive refractions

$$\frac{n_1}{p} + \frac{n_2}{i'} = \frac{n_2 - n_1}{r_1} \tag{4.8}$$

where r_1 is the curvature radius of the first surface. The image formed after the first refraction is the object of the second refraction, and its distance from the second surface is

$$p' = L - i' \tag{4.9}$$

so that the final image is formed at a distance i from the second surface given by

$$\frac{n_2}{L - i'} + \frac{n_1}{i} = \frac{n_1 - n_2}{r_2} \tag{4.10}$$

In the **thin lens approximation**, $L \to 0$ so that eq. (4.10) is reduced to

$$-\frac{n_2}{i'} + \frac{n_1}{i} = \frac{n_1 - n_2}{r_2} \tag{4.11}$$

To eliminate the intermediate image and arrive at a single relation between object and image distances, add the two equations (4.8) and (4.11):

$$\frac{n_1}{p} + \frac{n_1}{p} = (n_2 - n_1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{4.12}$$

4.3.2 The lensmaker's equation

Almost always, of course, the outside medium is air – $n_1 = 1$ – , and the material of the lens is glass, with a refractive index $n_2 = n$ depending on the particular type of glass used. In this case, eq.(4.12)

$$\frac{1}{p} + \frac{1}{i} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{4.13}$$

Notice that the right-hand side only depends on the characteristics of the lens: What it's made of, and the curvature radii of its surfaces. It has dimensions of $(\text{length})^{-1}$; and what is more, when the object is at infinity, so that the incident rays are parallel to the axis, the image is formed at a distance from the lens equal to the inverse of the right-hand side. All this indicates that we can define a **focal length** for the lens

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{4.14}$$

This is the **lensmaker's equation**. It tells a lensmaker what curvature radii he should achieve when he grinds a lens to obtain a desired focal length f, given that he's working with a particular type of glass of refractive index n.

4.3.3 The thin lens equation

When we substitute f from equation (4.14) in equation (4.12), we get the following very simple recipe for locating the image formed by a lens:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \tag{4.15}$$

We wil call this the **thin-lens equation**. It is identical to the mirror equation (3.2)! But beware: The sign convention is not the same.

4.3.4 Converging and diverging lenses

If the focal length is positive, the image of an object at infinity is formed by rays converging at a point behind the lens. Such a lens is called **converging**. On the other hand, if the focal length is negative, the rays from an object at infinity diverge after passing through the lens, appearing to come from a point somewhere in front of the lens. This is called a **diverging** lens.

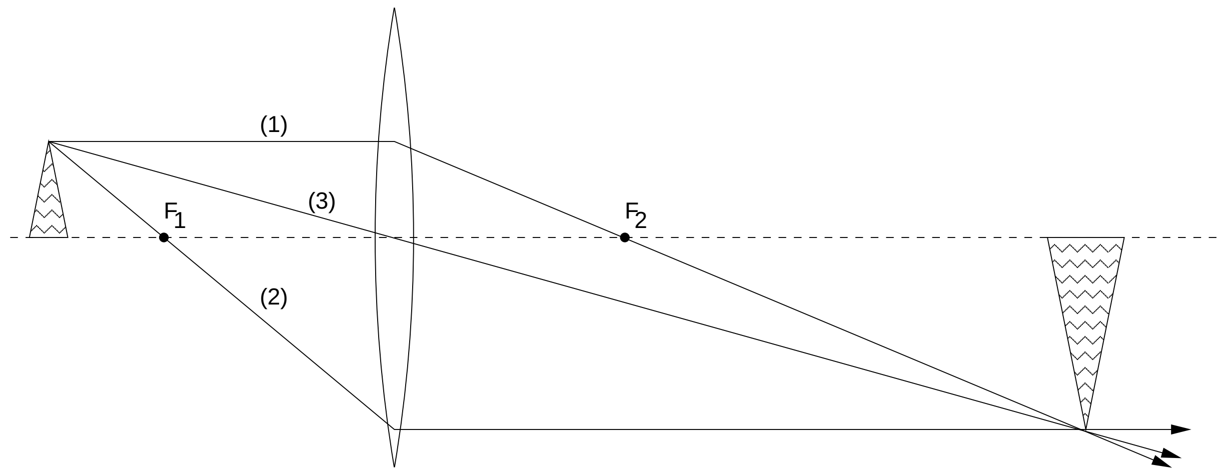

Figure 4.3: Ray tracing with a converging lens.

Exercise 4.3.1 *Use the lensmaker's equation to show that a converging lens is thicker in the middle, and a diverging lens is thinner in the middle.*

4.3.5 Ray tracing with lenses

For the purposes of ray tracing, every lens is said to have two focal points, a **primary focal point** and a **secondary focal point**. A *converging* lens has its primary focal point on the side from where the light is coming (usually drawn on the left), and the secondary focal point is symmetrically on the other side of the lens. The opposite is true of a *diverging lens.*

As with mirrors, we can locate an image formed by a lens graphically, with the help of three auxiliary rays (see Figures 4.3 and 4.4):

- A ray parallel to the axis passes through (or appears to pass through) the secondary focal point F_2. (Ray 1 in the figures).
- A ray passing through (or when extended, appearing to pass through) the primary focal point F_1 emerges from the lens parallel to the axis. (Ray 2 in the figures).
- A ray falling on the lens at its centre passes through undeflected. (Ray 3 in the figures).

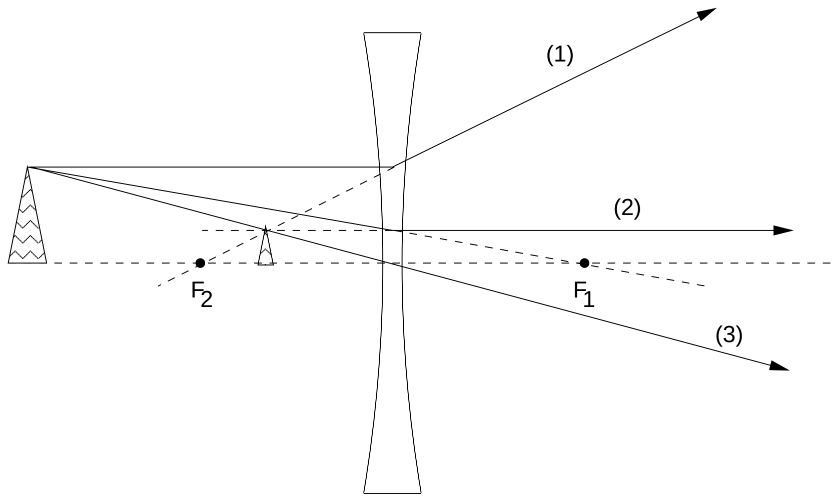

Figure 4.4: Ray tracing with a diverging lens.

4.3.6 Real and virtual images

In Figure 4.4 the image was formed at the intersection not of the light rays emerging from the lens, but of their extension backwards. This means that the image is virtual: It cannot be projected on a screen. In fact, the image is formed behind the lens, so if a screen were placed there, the light would be blocked and would not be able to pass through the lens at all!

It is possible to tell whether an image is real or virtual from the thin-lens equation, without having to locate it by ray tracing. In the case of Figure 4.4 the thin-lens equation is

$$\frac{1}{p} + \frac{1}{i} = -\frac{1}{|f|} \tag{4.16}$$

where we have emphasized that the focal length of the lens is negative by writing it as $-|f|$. Then the position of the image is

$$i = -\frac{p|f|}{p + |f|} < 0 \tag{4.17}$$

The image is virtual if and only if i is negative.

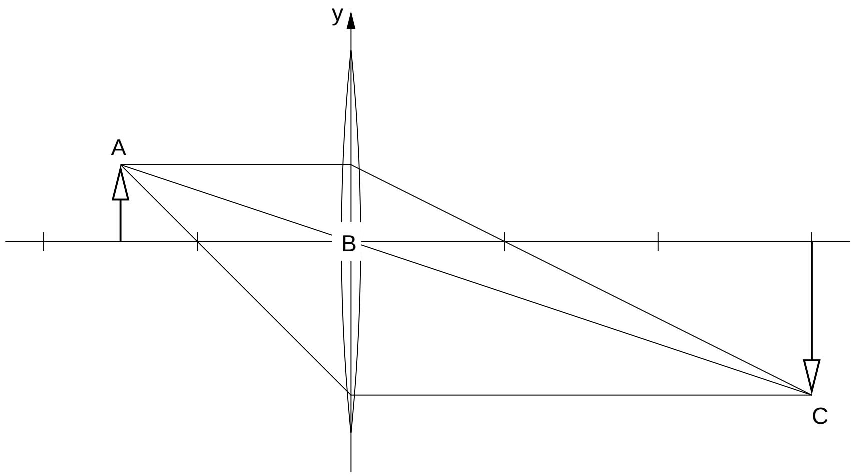

Figure 4.5: Lateral magnification by a lens.

4.3.7 Lateral magnification

Figure 4.5 illustrates how an image may be located by ray tracing. The optical axis has been marked off in units of the focal length f. Notice also that we have drawn a y axis, positive upwards. Clearly, the image is larger than the object, and also inverted. We can also get this information directly from the **lateral magnification**

$$m = \frac{y_i}{y_o} \tag{4.18}$$

where y_i is the height of the image and y_o is the height of the object. These heights are measured along the y axis, so in this case $y_o > 0$ but $y_i < 0$. In this way, the absolute value of m measures how much bigger (or smaller) the image is compared with the object, and the sign of m tells us whether the image is **upright** or **inverted** relative to the object.

It is evident from the figure, using ray ABC, that

$$m = -\frac{i}{p} \tag{4.19}$$

Given the position of the object p, from the thin-lens equation we can calculate i, and hence also the lateral magnification m. For example, in the case of the figure, if we let $f = 1$, then $p = 3/2$, so

$$\frac{2}{3} + \frac{1}{i} = 1 \Rightarrow i = 3 \Rightarrow m = -2 \tag{4.20}$$

which is confirmed by ray tracing.

Chapter 5

Optical instruments using lenses

5.1 Single-lens systems

To see how the analytical tools developed in the previous chapter may be applied to the design of some simple optical systems, we study first systems formed by a single lens. You may find it useful to reproduce these examples using our virtual optical bench.

5.1.1 A magnifying glass

Angular size

What we perceive as the "size" of an object is the angle that it subtends in our field of vision. (See Figure 5.1). Clearly, to increase the angular size of a small object in order to see it better, we need

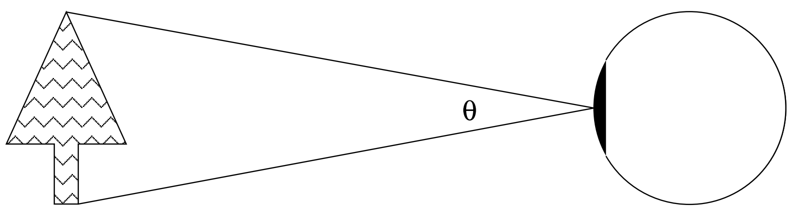

Figure 5.1: Angular size

to bring it as close to the eye as possible. But there is a limit to how close we can bring it: Beyond a certain distance, called the **near-point distance**, we can no longer focus the eye to create a sharp image on the retina. A magnifying glass is a converging lens which creates an image of an object very close to the eye at the near point, or slightly beyond it, so that the image may be seen sharply in focus.

Since the image is formed behind the lens, it is a virtual image. A ray-tracing analysis of the magnifying glass is shown in Figure 5.2. If the height of the original object is y, its angular size in the small-angle approximation is essentially the same as the tangent of the angle,

$$\theta = \frac{y}{p} \tag{5.1}$$

The eye is most relaxed when it is focused at infinity, so we want to form the image with the glass as far away as possible. This means that p must be very slightly under the focal length f, so we may write equation 5.1 as

$$\theta = \frac{y}{f} \tag{5.2}$$

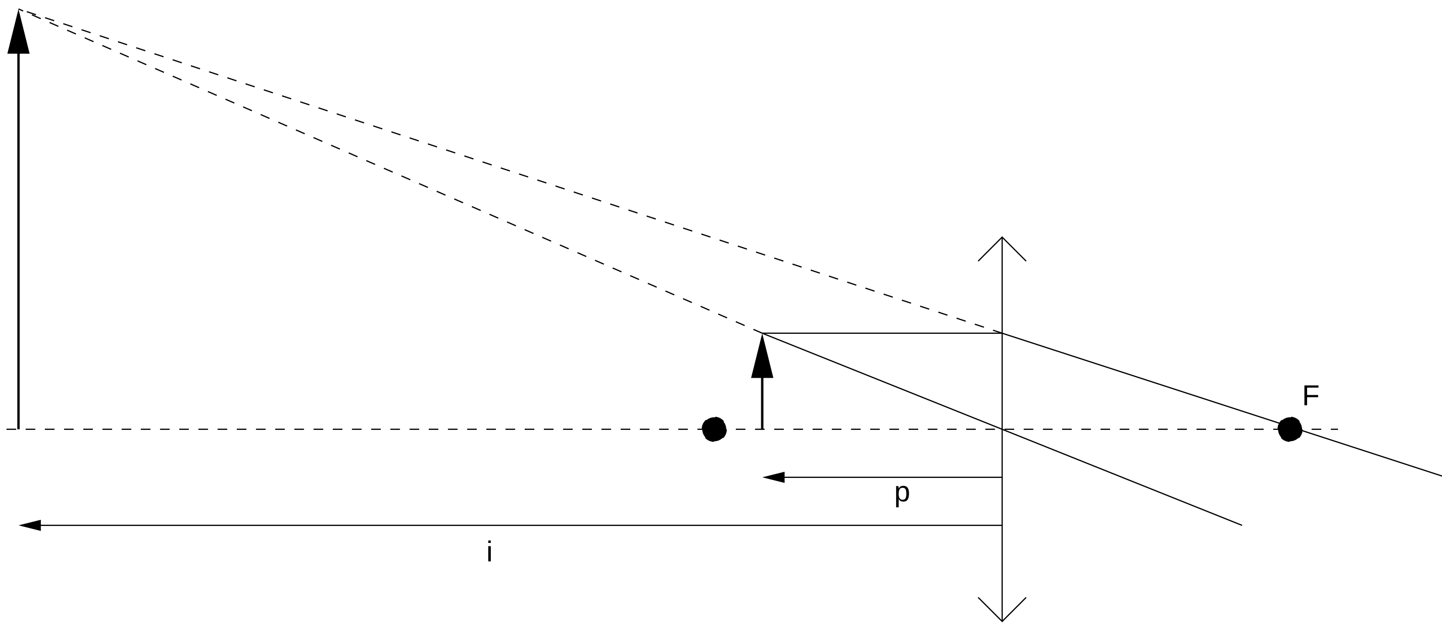

Figure 5.2: Image formation by a magnifying glass

and this is also the angular size of the image.

If we were obliged to look at the object at the near point distance d_N with the naked eye, its angular size would be

$$\theta' = \frac{y}{d_N} \tag{5.3}$$

so the angular magnification of the magnifying glass is

$$m_\theta = \frac{\theta}{\theta'} = \frac{d_N}{f} \tag{5.4}$$

Clearly, a magnifying glass should have a small focal length in comparison with d_N, which is normally estimated as 25 cm.

5.2 Compound optical systems

Many useful instruments consist of two or more lenses aligned on a common axis. In this section we will discuss two-lens systems. The same ray-tracing techniques, and the same thin-lens formulas may be applied, bearing in mind that the image formed by the first lens becomes an object for the second lens. Figure 5.3 shows such a system schematically. The first image is formed at a distance

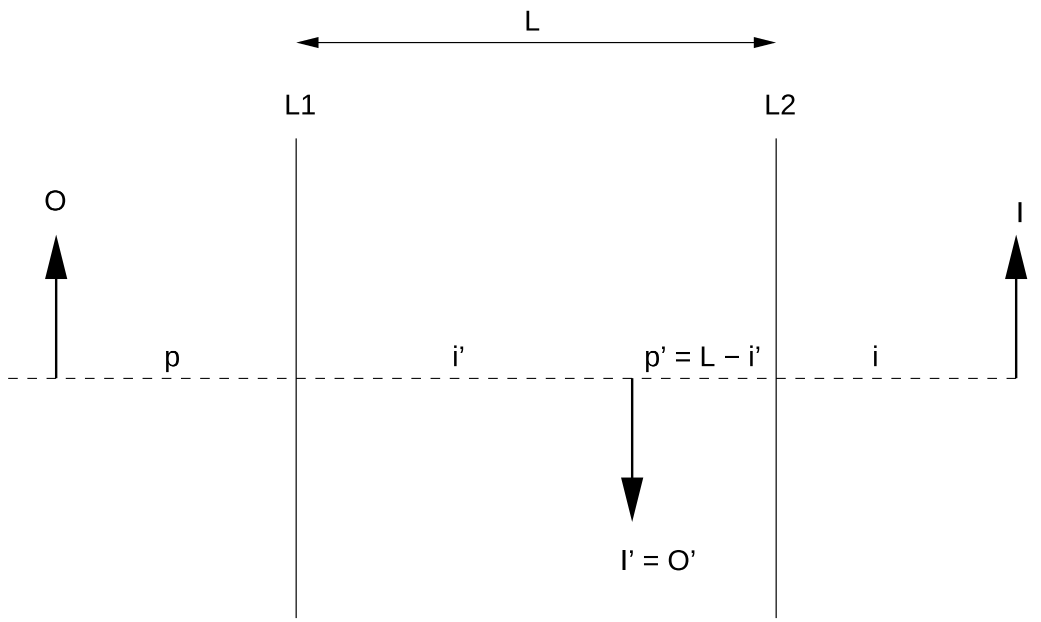

Figure 5.3: A two-lens system

i' from the first lens L_1 given by

$$\frac{1}{p} + \frac{1}{i'} = \frac{1}{f_1} \tag{5.5}$$

and as an object for the second lens, its object distance is $p' = L - i'$. The final image is formed at a distance i from the second lens, given by

$$\frac{1}{L - i'} + \frac{1}{i} = \frac{1}{f_2} \tag{5.6}$$

Eliminating i' between these two equations, a single equation may be obtained relating i, p, and the two focal lengths f_1 and f_2.

As a practical application of a two-lens system, we will discuss one particular instrument here, the refracting telescope.

5.3 The refracting telescope

Although real refracting telescopes have complex lens combinations to correct the image, for the purpose of understanding how they work it is sufficient to regard a telescope as consisting of two elements, the **objective** and the **eyepiece**, or **ocular**.

Figure 5.4 shows how an image is formed by a telescope. The object is very distant, and the objective forms an image of it at an image distance equal to its focal length. The eyepiece is set up so that its focal point practically coincides with that of the objective, so that the intermediate image will form an image at infinity as shown. But because the focal length of the eyepiece is smaller, the angular size of the final image is larger than the angular size of the object.

To calculate the angular magnification of the telescope, $m_\theta = \theta'/\theta$, we note first that

$$\theta = \angle BAD \approx \tan \angle BAD = \frac{h}{f_o} \tag{5.7}$$

Here f_o is the focal length of the objective, and h is the height of the image formed by the objective. Notice how the sign conventions apply here: h is negative because the image is oriented downwards, so $\theta = h/f_o$ is also negative, since $f_o > 0$ for a converging lens. This is consistent with the convention that angles are counted as positive going counterclockwise, so the angle θ' from the optical axis to the light ray is negative.

As for θ',

$$\theta' = \angle BCD \approx \tan \angle BCD = -\frac{h}{f_e} \tag{5.8}$$

Here f_e is the focal length of the eyepiece. The minus sign is necessary to make θ positive, because $h < 0$.

Now we can calculate the angular magnification as

$$m_\theta = \frac{\theta'}{\theta} = -\frac{f_o}{f_e} \tag{5.9}$$

The telescope forms an inverted image, which is sometimes undesirable. The **spyglass**, or **terrestrial telescope**, is used to observe objects closer to the observer. It is a variation on the telescope which produces an upright image. The essential difference is that the eyepiece is a *diverging lens*. Figure 5.5 shows the paths of the rays in this case.

Notice that the intermediate image formed by the objective lens falls to the right of the eyepiece. When this happens, this image is said to be a **virtual object** for the eyepiece. All this means is

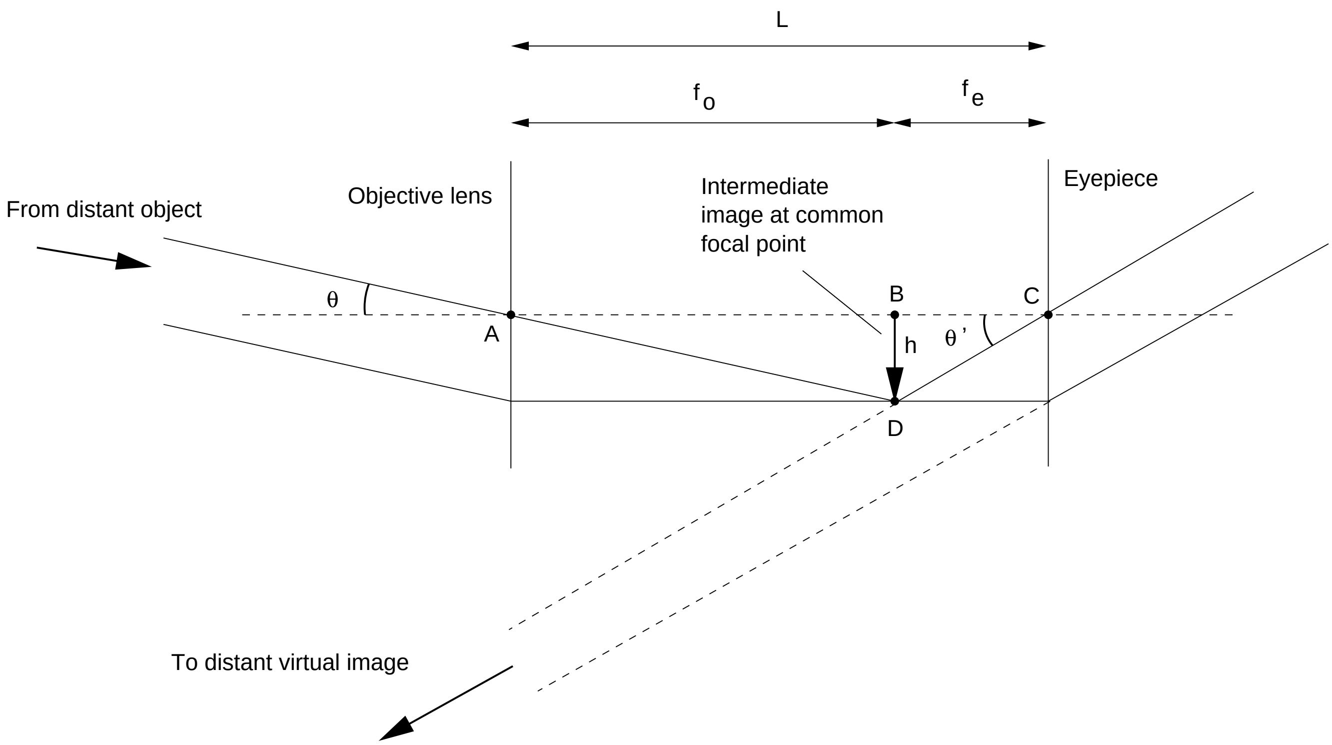

Figure 5.4: A refracting telescope.

that an image would be formed there if the eyepiece didn't exist. For the purpose of calculation, the object distance for the eyepiece is negative.

Remarkably, equations (5.7) and (5.8) still hold, so the angular magnification is still given by equation (5.9). But, since $f_e < 0$, the angular magnification is positive, which means that the final image is upright.

Exercise 5.3.1 *Verify that equations (5.7) and (5.8) still hold even for the spyglass. Pay special attention to the signs of the angles. Remember that in this case* $f_e < 0$*.*

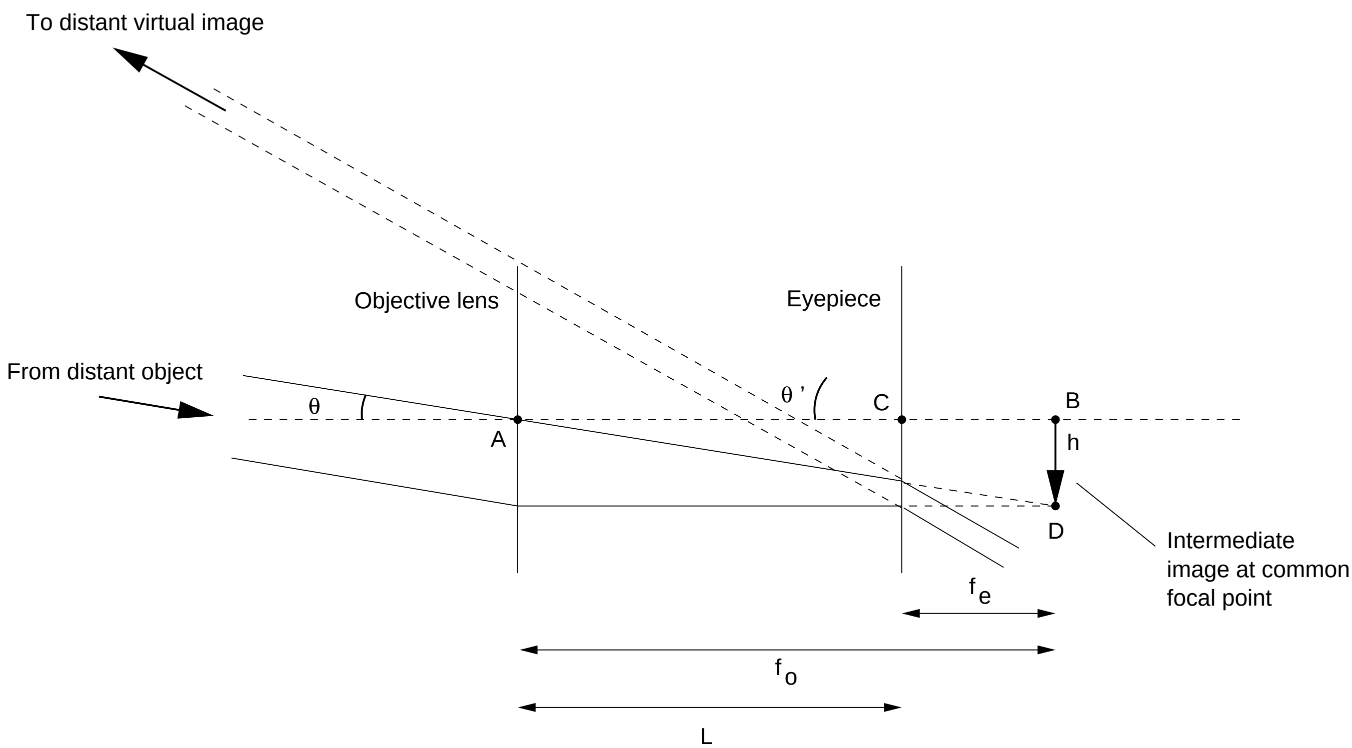

Figure 5.5: A spyglass.

Chapter 6

Interference and diffraction

6.1 Wave phenomena

6.1.1 Introduction

Geometrical optics allows us to understand and even to design a wide variety of optical instruments. Certain optical phenomena, however, can only be explained in terms of the wave properties of light. In this chapter we focus on two such phenomena, **interference** and **diffraction**.

6.1.2 Interference

Thomas Young (1773 – 1829) performed a now classical experiment that showed up the wave properties of light. By splitting up a light beam into two, he effectively created two very close, **coherent** sources of light, emitting waves of identical wavelength and in step with each other. As the wavefronts spread out in space, they combined with each other, producing interference. At certain locations, the two waves would arrive in step and enhance each other; at certain other locations, the waves would arrive exactly out of step and cancel each other. This simulation shows interference produced in a "ripple tank" with surface waves in a liquid.

Location of interference maxima

We limit our analysis to the formation of interference patterns very far from the sources. Consider the situation illustrated in Figure 6.1. Two rays meeting at a distant point with direction θ with respect to the symmetry axis of the two sources, will interfere destructively or constructively depending on the **optical path difference** (OPD in the figure). If $\mathrm{OPD} = m\lambda$, where m is a whole number, then the waves from the two sources will arrive in step where they meet, and interfere constructively, creating a bright point of light at that location. From the figure we see immediately that the possible directions of interference maxima are given by

$$d \sin\theta = m\lambda \tag{6.1}$$

where $m = 0, \pm 1, \pm 2, \ldots$.

6.1.3 Diffraction

More intriguing perhaps is that we observe bright and dark bands even when light passes through a *single slit*. This is because accoridng to Huygens' principle, each point along the aperture acts as

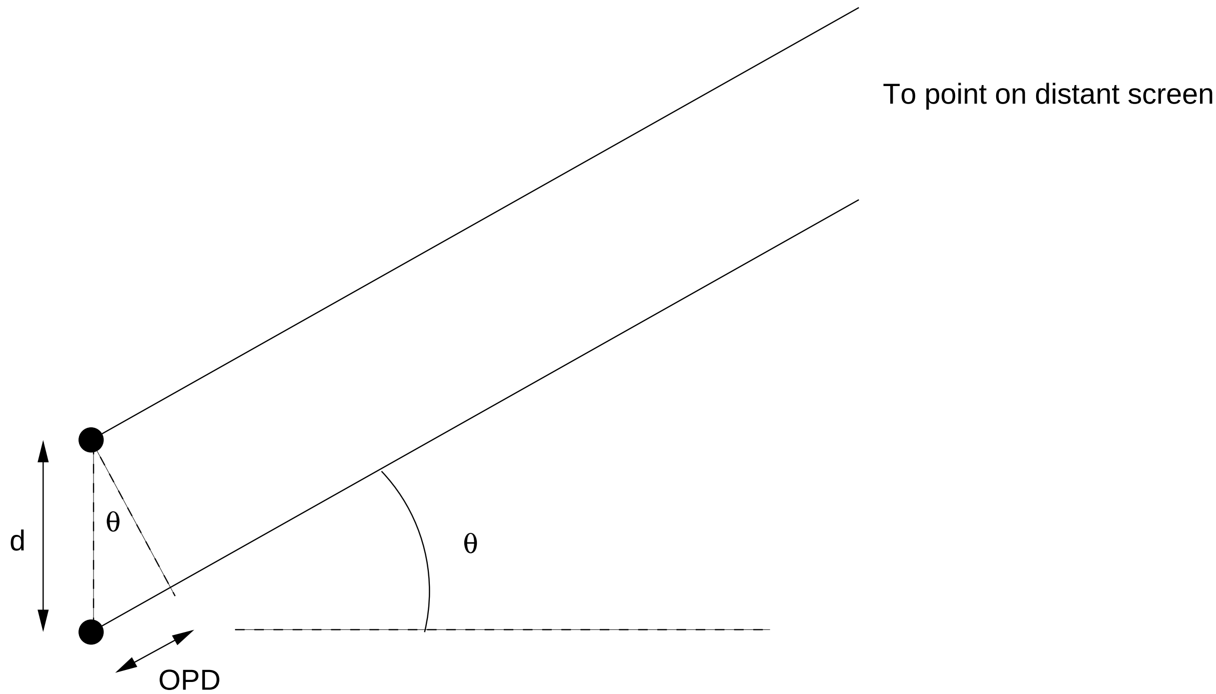

Figure 6.1: Two-slit interference

a new source of circular wave fronts, all of which will combien to produce an interference pattern far away. (See Figure 6.2).

Location of diffraction minima from a single slit

Of all the countless point sources along the aperture, consider a particular pair: One point on the edge of the aperture, and another exactly halfway across the aperture, as shown in Figure 6.3. These will interfere destructively if the OPD = $\lambda/2$. Similarly, all other pairs of points across the aperture separated by $a/2$ will also interfere destructively. So the first dark fringe will be formed in a direction given by

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \tag{6.2}$$

or more simply

$$a\sin\theta = \lambda \tag{6.3}$$

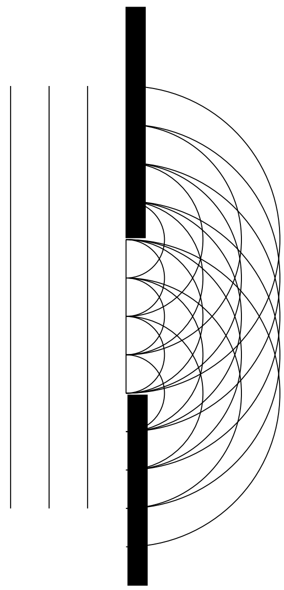

Figure 6.2: Waves emerging from an aperture.

The next minimum is formed when pairs of sources separated by $a/4$ interfere destructively, i.e. when

$$\frac{a}{4}\sin\theta = \frac{\lambda}{2} \tag{6.4}$$

that is, when

$$a\sin\theta = 2\lambda \tag{6.5}$$

In general, diffraction minima may be found in the directions given by

$$a\sin\theta = m\lambda \tag{6.6}$$

where $m = \pm 1, \pm 2, \ldots$.

Diffraction by a circular aperture

A slit will produce a diffraction pattern consisting of bright and dark fringes parallel to the slit. Different shape apertures will produce accordingly different shape diffraction patterns. For example, a circular aperture produces a very bright central spot, surrounded by alternating birght and dark

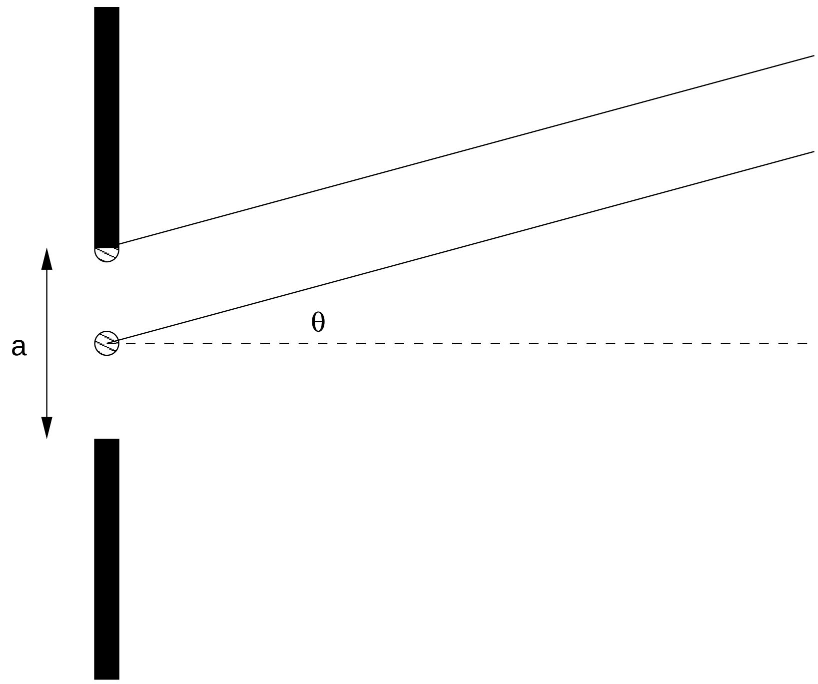

Figure 6.3: Formation of diffraction minima by a single slit.

rings. (See this image). A more complex calculation shows that the direction of the first dark ring, which is practically the same as the edge of the central spot, is given by

$$\sin\theta = 1.22\frac{\lambda}{d} \tag{6.7}$$

Resolution of an optical instrument

Any optical instrument gathers light through an aperture, so a poitn source of light will be imaged not as a point, but as a diffraction pattern, of which most is contained in the central spot. This limits the **resolution** of the instrument, that is, the ability to distinguish sufficient detail. For example, a telescope may not be able to distinguish two stars that are very close together, because their diffraction patterns overlap. (See this image). **Rayleigh's criterion** states that two points may not be resolved if their angular separation is less than

$$\theta_R = \arcsin\left(\frac{1.22\lambda}{d}\right) \approx \frac{1.22\lambda}{d} \tag{6.8}$$

Thus the bigger the aperture of a telescope, the better is its resolving power. (Also, it gathers more light so it is capable of registering fainter objects than a smaller instrument). Another good example of the use of a large aperture to improve resolution is the unusually large eye of predatory birds like eagles or owls.

6.1.4 Diffraction gratings

One of the most important applications of interference is **spectrometry**, based on the interference pattern produced not by one or two, but by very many thin slits close together. Such devices are called **diffraction gratings**. Interference maxima will be produced in the same directions as with only two slits. Thus, if the separation between adjacent slits is d, the maxima will be in the directions

$$d \sin\theta = m\lambda \tag{6.9}$$

where $m = 0, 1, 2, \ldots$.

The important feature of a diffraction grating is that the interference maxima will not be in the form of broad bands but rather very thin, bright lines. We can show this by calculating the **half-width** of the central maximum. (Figure 6.4 shows what we mean by half-width). The brightness of the line drops to zero in the direction in which the N slits have an interference minimum. As N is usually a very large number, the situation is very similar to diffraction by a single slit of width Nd. So the first interference minimum is in the direction $\Delta\theta_{hw}$, given by

$$(Nd)\sin\Delta\theta_{hw} = \lambda \tag{6.10}$$

and in the small-angle approximation

$$\Delta\theta_{hw} = \frac{\lambda}{Nd} \tag{6.11}$$

In general, for any order of interference m,

$$\Delta\theta_{hw} = \frac{\lambda}{N\cos\theta\ d} \tag{6.12}$$

Spectrometry: Application of a diffraction grating

All atoms and molecules are capable of emitting electromagnetic radiation when they absorb energy, at very distinct wavelengths, characteristic of each particular substance. Thus the "emission spectrum", as the set of wavelengths of the emitted light is called, is an important analytical tool.

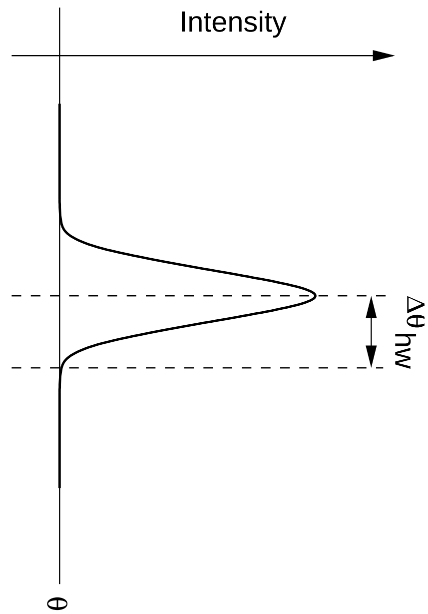

Figure 6.4: The half-width of the central interference line produced by a diffraction grating.

The light emitted by a sample of a substance may be split up effectively by passing it through a diffraction grating, since the direction of each interference maximum depends on wavelength. More than one complete spectrum may be formed in principle, one for each order of interference.

An interesting variation on this theme is when a relatively cool gas is illuminated by light with a broad continuous spectrum (such as the uppermost layers of a star). In this case, the gas absorbs light at characteristic wavelengths, leaving dark lines in the resulting spectrum. Again, the chemical composition of this gas is revealed by which wavelengths are absent from the original continuous spectrum.

For example in astrophysics the chemical composition of distant objects may be revealed by spectrometry.

Dispersion and resolving power of a diffraction grating

We use a diffraction grating to measure the wavelength of a light emission by measuring the direction in which a bright line is formed. In practice, an uncertainty in the direction of the bright line will result in an uncertainty in the wavelength. According to the error propagation formula,

$$\Delta\lambda = \left|\frac{d\lambda}{d\theta}\right| \Delta\theta \tag{6.13}$$

Two properties of the grating contribute to $\Delta\lambda$. One is the **dispersion** of the grating,

$$D = \left|\frac{d\theta}{d\lambda}\right| \tag{6.14}$$

and the other is the minimum separation $\Delta\theta$ between two lines that the grating can **resolve**. Now we have seen that the interference maxima occur at angles θ given by

$$d \sin\theta = m\lambda \tag{6.15}$$

where $m = 0, \pm 1, \pm 2, \ldots$. Therefore

$$D = \frac{m}{d\cos\theta} \tag{6.16}$$

Clearly to reduce the uncertainty $\Delta\lambda$ we want a high value of the dispersion. A small d is helpful.

The other contribution to the uncertainty in λ is that lines have a finite half-width. So if two lines are very close in wavelength, so that their separation is less than their half-width, they will overlap. To estimate the minimum $\Delta\lambda$ that the instrument can **resolve**, we'll substitute $\Delta\theta_{hw}$ from equation (6.12):

$$\Delta\lambda = \frac{dcos\theta}{m}\frac{\lambda}{N\cos\theta d} = \frac{\lambda}{mN} \tag{6.17}$$

The **resolution** of the grating is defined as

$$R = \frac{\lambda}{\Delta\lambda} = Nm \tag{6.18}$$

The higher the resolution, the smaller the uncertainty $\Delta\lambda$. Increasing the number of slits will do the trick.

6.2 Summary of formulas in this chapter

Two-slit interference: The directions of interference *maxima* are given by

$$d \sin\theta = m\lambda \tag{6.19}$$

where $m = 0, \pm 1, \pm 2, \ldots$. d is the separation between the slits.

Diffraction by a single slit: The directions of diffraction *minima* are given by

$$a \sin \theta = m\lambda \tag{6.20}$$

where $m = \pm 1, \pm 2, \ldots$. a is the width of the slit.

Diffraction by a circular aperture: The angular half-width of the central diffraction spot is given by

$$\sin \theta = 1.22 \frac{\lambda}{d} \tag{6.21}$$

Rayleigh's criterion: Two points on an object may be resolved by an optical instrument of aperture diameter d in light of wavelength λ if their angular separation is at least

$$\theta_R = \arcsin \left(\frac{1.22\lambda}{d} \right) \approx \frac{1.22\lambda}{d} \tag{6.22}$$

Diffraction grating: A diffraction grating produces bright lines in directions θ given again by

$$d \sin \theta = m\lambda \tag{6.23}$$

where $m = 0, \pm 1, \pm 2, \ldots$.

The half-width of a line is

$$\Delta\theta_{hw} = \frac{\lambda}{Nd \cos \theta} \tag{6.24}$$

where N is the number of slits in the grating.

The dispersion of the grating is

$$D = \frac{m}{d \cos \theta} \tag{6.25}$$

and the resolution is

$$R = \frac{\lambda}{\Delta\lambda} = Nm \tag{6.26}$$

Appendix A

Small angle approximation

A.1 Small angle approximation

For small values of θ, the functions $\sin\theta$ and $\tan\theta$ take on particularly simple forms. Consider a very "thin" right triangle, as shown in Figure A.1. Since $a \approx h$,

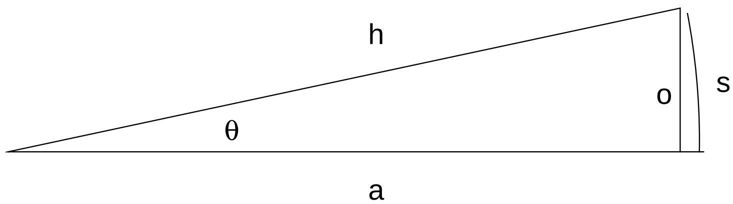

Figure A.1: A thin right triangle

$$\sin\theta \approx \tan\theta \tag{A.1}$$

Also, to a very good approximation the triangle resembles a circular wedge, with o replaced by the arc s, so that $o/h \approx s/h = \theta$ (if θ is measured in radians). Putting everything together,

$$\theta \approx \sin\theta \approx \tan\theta \tag{A.2}$$

Appendix B

Derivations for the exam

B.1 Derivations for the Module 7 exam

1. **Snell's Law**: Two transparent media (call them 1 and 2) are separated by a plane interface. Waves travel in each medium with speed v_1 or v_2. A plane wavefront propagating in medium 1 reaches the interface at an angle θ_i, and propagates into medium 2 at a different angle θ_r with respect to the interface. Show that

$$\frac{\sin\theta_i}{v_1} = \frac{\sin\theta_r}{v_2}$$

Answer: See Figure B.1. The line PR is part of the incident wave front, and QS is part of the refracted wave front. In a time t, point P propagates to Q in medium 1. The distance PQ is $v_1 t$. In the same time, point R propagates inside medium 2 to another point S, a distance $v_2 t$ from R. In fact, because Q and S are on the same wave front, S must be where it is shown in the figure. In triangle PQR, we have that

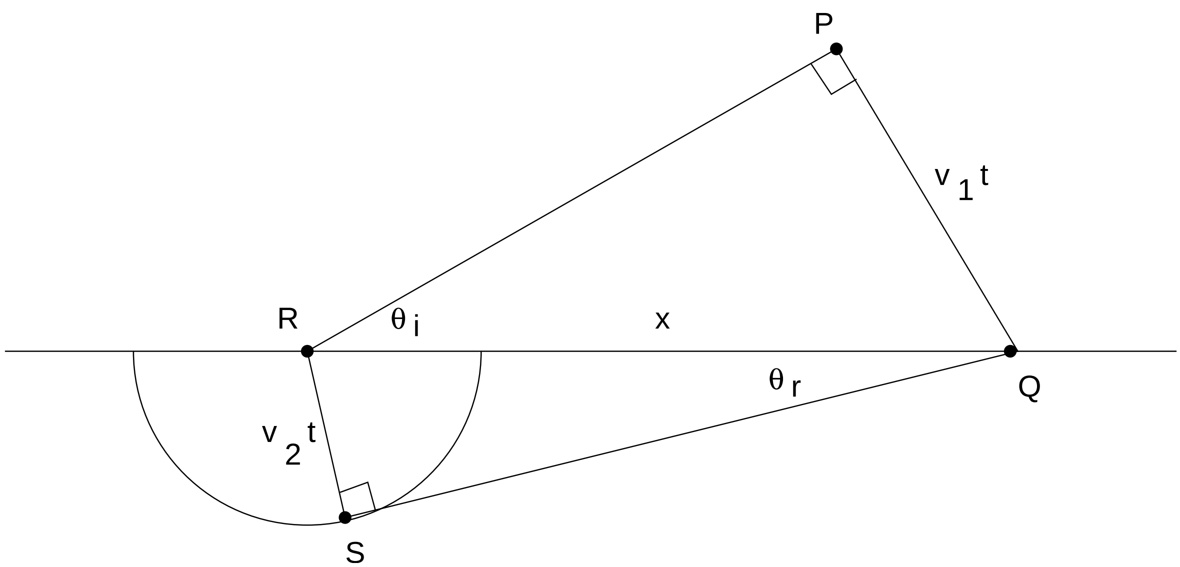

Figure B.1: Derivation of the law of refraction.

$$\sin\theta_i = \frac{v_1 t}{x} \tag{B.1}$$

and in triangle QRS,

$$\sin\theta_r = \frac{v_2 t}{x} \tag{B.2}$$

and therefore

$$\frac{\sin\theta_i}{v_1} = \frac{\sin\theta_r}{v_2} \tag{B.3}$$

2. **Mirror equation**: Derive the mirror equation

$$\frac{1}{i} + \frac{1}{p} = \frac{1}{f}$$

where i is the image distance, p is the object distance, and f is the focal length of the mirror.

Answer: See Section 3.4. Please note that all the steps must be completed explicitly, including the notes say "after some algebraic manipulation...".

3. **Young's experiment**: Light of wavelength λ illuminates two thin slits, separated by a distance d. On a distant screen an interference pattern is produced. Define an axis from the slits to the central maximum. Show that every other maximum lies in a direction at an angle θ_m with respect to this axis, given by

$$d\sin\theta_m = m\lambda$$

where $m = 0, \pm 1, \pm 2, \ldots$.

Answer: See section 6.1.2.